Steady-State FE Modelling of Induction Machines: the Harmonic-Balance Approach versus the Classical Fundamental-Frequency Phasor Approach

J. Gyselinck[1], Y. Mollet[1] and R. V. Sabariego[2]

[1] BEAMS Department, Université Libre de Bruxelles (ULB), 1050 Bruxelles, Belgium. E-mail: johan.gyselinck@ulb.ac.be
[2] Department of Electrical Engineering, KU Leuven, Campus EnergyVille, 3600 Genk, Belgium

Abstract **— In this paper three different approaches for the steady-state finite-element (FE) modelling of induction machines (IMs) are discussed and compared regarding their (spectral) accuracy and computational cost; to this end a 3-phase 3 kW squirrel-cage IM with sinusoidal voltage supply is considered. The reference approach is plain time-stepping; its drawback is the possibly very long transient and time interval to traverse. The well-known phasor approach, which involves only the fundamental stator and rotor frequency, is much quicker but may be insufficiently accurate. In the Harmonic Balance (HB) method, two sets of frequencies are to be fixed a priori, for stator- and rotor-side variables respectively, with a view to an adequate compromise between accuracy and computational cost.**

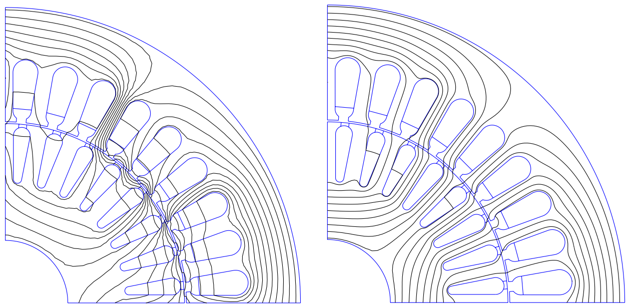

Fig. 1. 3kW induction machine: geometry (1 pole) and flux lines (real part) obtained with the phasor approach, at 500 rpm (left) and 1425 rpm (right)

I. TIME-STEPPING VERSUS PHASOR ANALYSIS

Time-stepping 2D FE analysis of electromagnetic devices, and in particular of rotating electrical machines, is common practice since, say, two to three decades [1]. Such analysis comprises: rotation (and associated harmonics), magnetic saturation (and possibly hysteresis), electrical circuit coupling and voltage supply, skew, faults, etc. In case one is only interested in the steady-state solution, for given speed and supply conditions, the possibly very long transient to traverse may be prohibitive. Also, in case of faults, e.g. broken bars and (dynamic) eccentricity in Squirrel-Cage (SC) IMs, high-resolution spectral analysis and thus a long simulation interval may be required [2].

In this short paper, we will consider only healthy IMs, and in particular a 4-pole 3 kW 380/220 V 50 Hz SCIM [2]. It has a single-layer 36-slot stator winding and 32 rotor slots, see Fig. 1. With $f_{\mathrm{s},1} = 50\,\mathrm{Hz}$ supply, at speed N (in rpm), slip and fundamental rotor frequency are $s = (1500 - N)/1500$ and $f_{\mathrm{r},1} = s f_{\mathrm{s},1}$. The first stator harmonics due to rotor slotting are $f_{\mathrm{s},k_\mathrm{s}} = f_{\mathrm{s},1} + k_\mathrm{s}\, 32N/60$, with $k_\mathrm{s} = \pm 1$; conversely, the first rotor harmonics due to stator slotting are $f_{\mathrm{r},k_\mathrm{r}} = f_{\mathrm{r},1} + k_\mathrm{r}\, 12N/60$, with $k_\mathrm{r} = \pm 1$. These frequencies are generally not a multiple of $f_{\mathrm{s},1}$, except at synchronous speed ($s = 0$, [3]); this may be a major complication for accurate spectral analysis. The cases $s = 0$ and $s = 2/3$ are detailed in the next section.

The phasor approach consists in solving one system of complex-valued equations at the stator supply frequency and for a certain (arbitrary) rotor position, whereby nonzero speed is emulated by dividing the cage resistivity by the corresponding non-unitary slip [1, 4]. Magnetic saturation is considered through an equivalent BH-curve, the choice of which can considerably affect the results [5]. As harmonics due to rotation are the focus of this paper, magnetic saturation will be mostly disregarded hereafter.

Fig. 1 shows two sets of flux lines obtained with the phasor approach, at rated supply voltage. One such linear computation (with 4460 complex-valued unknowns and equations) takes around 0.16 CPUs on a 2.6 GHz Intel Core i7 and 16 Gb RAM Mac BookPro, using the open-source ONELAB/Gmsh/GetDP software [6].

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Both phasor and time-stepping computations have been done for a series of speeds from standstill up to synchronous speed. Stator current (amplitude of the fundamental 50 Hz component) and torque (average value) versus speed curves are shown in Fig. 2. The speed interval from breakdown speed (at maximum torque, around 1000 rpm) up to synchronous speed is the most important one in practice; the other interval is mostly relevant for the starting phase.

In case of time-stepping, by applying the supply voltages gradually, the transient can be considerably shortened, such that only, say, 10 supply periods (each of 20 ms) really need to be time-stepped [3]. One time-stepping simulation of 20×400 steps takes around 400 CPUs, i.e. 0.05 CPUs per time step; the number of periods and the number of steps per period can be reduced however. Fig. 2 also shows the influence of the window length of the Fourier analysis following the time-stepping, either 20 ms and 100 ms, with more or less smooth curves.

As for the phasor computations, Fig. 2 shows the influence of the rotor position along with some 3-phase asymmetry (and in particular the existence of an inverse current component, which is significant at high slip); the direct current is quasi rotor-position independent though. Overall the results obtained with the phasor approach are satisfactory at low slip (up to breakdown slip); at high slip, a very significant deviation from the reference time-stepping results can be observed, especially for the torque.

II. HARMONIC BALANCE

In [3] the HB approach is applied to the noload operation (at synchronous speed) of the same 3 kW SCIM. Up to 9 frequencies, all multiples of 50 Hz, are considered: up to 4 in the stator, and up to 5 in the rotor (including the dc term, i.e. 0 Hz). The HB results presented in [3], i.e. (stator and rotor) current and (stator and rotor) flux-density waveforms, are shown to converge well to the plain time-stepping results. This means that saturation and stator and rotor slotting are considered in the HB method as accurately as they are in a time-stepping simulation provided that a sufficient number of frequencies is considered.

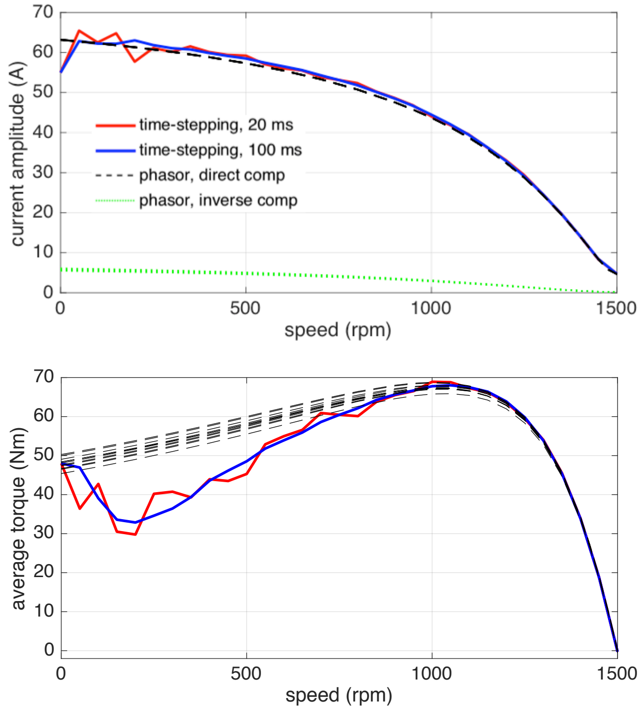

Fig. 2. Fundamental stator current amplitude (up, direct and inverse component) and average torque (down) versus speed curves obtained with the phasor approach, with 11 different rotor positions (spread evenly over the representative interval of 3.75 mech. degrees), and with time-stepping (Fourier analysis on the last 20 ms or 100 ms of the 400 ms simulation interval)

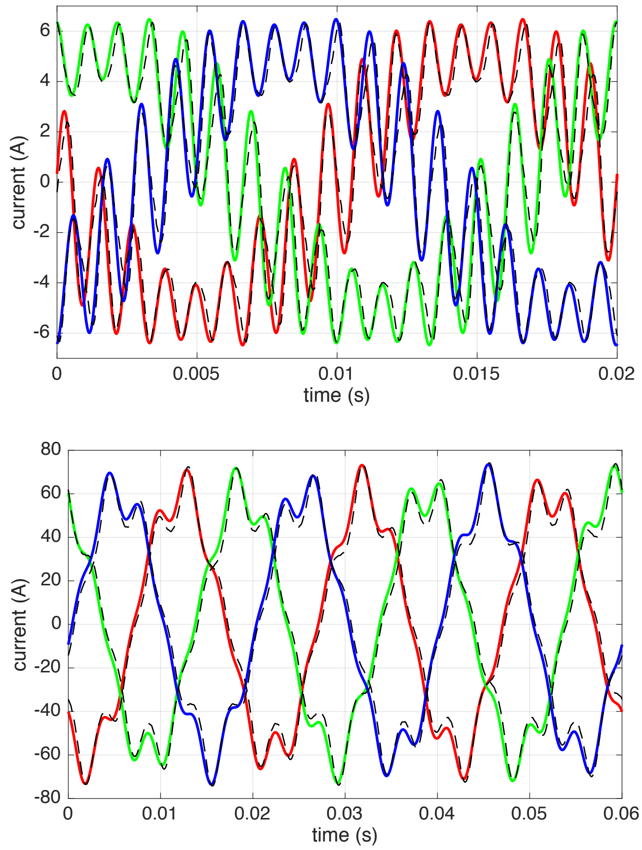

Fig. 3. Steady-state stator current waveforms at 1500 rpm (up, 20 ms) and 500 rpm (down, 60 ms), obtained with time-stepping (full thick colored lines) and with the HB method (dashed thin black lines)

The originality of the HB approach in [3], and more recently in [7], resides in the straightforward and unified way in which the set of HB equations are obtained, namely through the application of the Galerkin method to the time dimension, i.e. the cosine/sine functions (plus the unit function for the dc term, if relevant) are considered as both basis and weighing functions. In particular for movement, this is an important practical simplification compared to some other HB methods found in literature [8]; in case of the 2D model of a rotating machine, it amounts to a straightforward manipulation of the reluctivity matrix of the moving band, considering a number of discrete rotor positions within a fundamental period [3]. Reference [7] shows that the HB method allows for frequency-dependent material characterisation, which may be a decisive advantage compared to time-stepping.

Fig. 3 shows the stator-current waveforms at 1500 rpm and 500 rpm, obtained with the HB approach and agreeing very well with the time-stepping results. As for the first speed, the HB computation involves $\{50, 750, 850\}$ Hz on stator side and $\{0, 300, 600, 900\}$ Hz on rotor side; stator frequencies $\{150, 250\}$ Hz due to saturation are ignored here [3].

At 500 rpm, the considered frequencies are $\{50, 216.67, 316.67\}$ Hz and $\{33.33, 66.67, 133.33\}$ Hz, in agreement with the discussion in the beginning of the previous section, for a total computation time of around 26 CPUs. The fundamental frequency, period and rotor-position interval are 16.67 Hz, 60 ms, and 180 mech. degrees. The latter interval is sampled for the moving band contribution with a 1 mech. degree step; the number of positions considerably affects the CPU time of the assembly of the system of equations (here 11 CPUs out of 26 s), whereas the assembly phase is rather negligible CPU-time-wise in case of time-stepping.

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